PERT analysis

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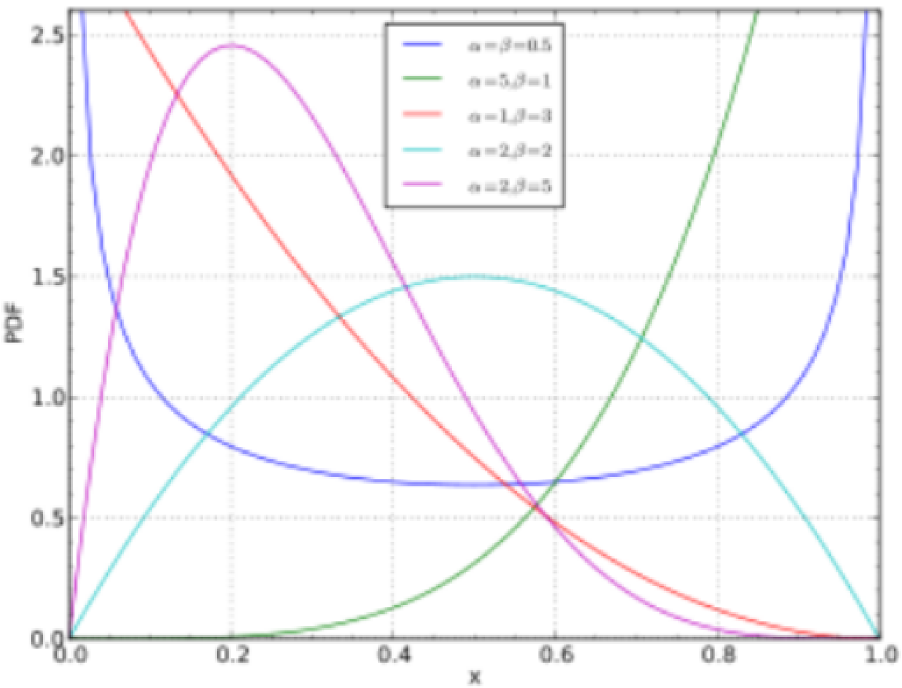
PERT (Project Review and Evaluation Technique)

Activity durations are random variables D_i with beta distribution

↳ a beta probability density is nonzero only on an interval $[a, b]$

↳ beta distributions have been observed experimentally

$a = 0, b = 1 \Rightarrow$



Modeling of durations

For each activity A_i the program manager estimates

a_i = shortest activity duration (most favorable prediction)

m_i = most likely activity duration

b_i = longest activity duration (least favorable prediction)

Mean and variance of D_i are approximated by

$$\mu_i = \frac{a_i + 4m_i + b_i}{6} \qquad \sigma_i^2 = \left(\frac{b_i - a_i}{6}\right)^2$$

Distribution of the shortest project duration

Standing assumption: durations D_i are independent random variables

PERT algorithm

1) Compute a critical path C in an AOA project model using CPM with $d_i = \mu_i$
2) Let $D_C = \sum_{A_i \in C} D_i$ be the random variable describing the total duration of C. Then, D_C has

mean: $\mu_C = \sum_{A_i \in C} \mu_i$

variance: $\sigma_C^2 = \sum_{A_i \in C} \sigma_i^2$

Project analysis

- If C is composed by several activities, from the central limit theorem, D_c is almost a Gaussian random variable with mean μ_c and variance σ_c^2
- For $\alpha \in (0,1)$, the $(1-\alpha)\%$ confidence interval for D_c is

$$I_\alpha = [\mu_c - z_\alpha \sigma_c, \mu_c + z_\alpha \sigma_c]$$

where $z_\alpha : F(z_\alpha) = 1 - \frac{\alpha}{2}$ and F is a standard Gaussian distribution

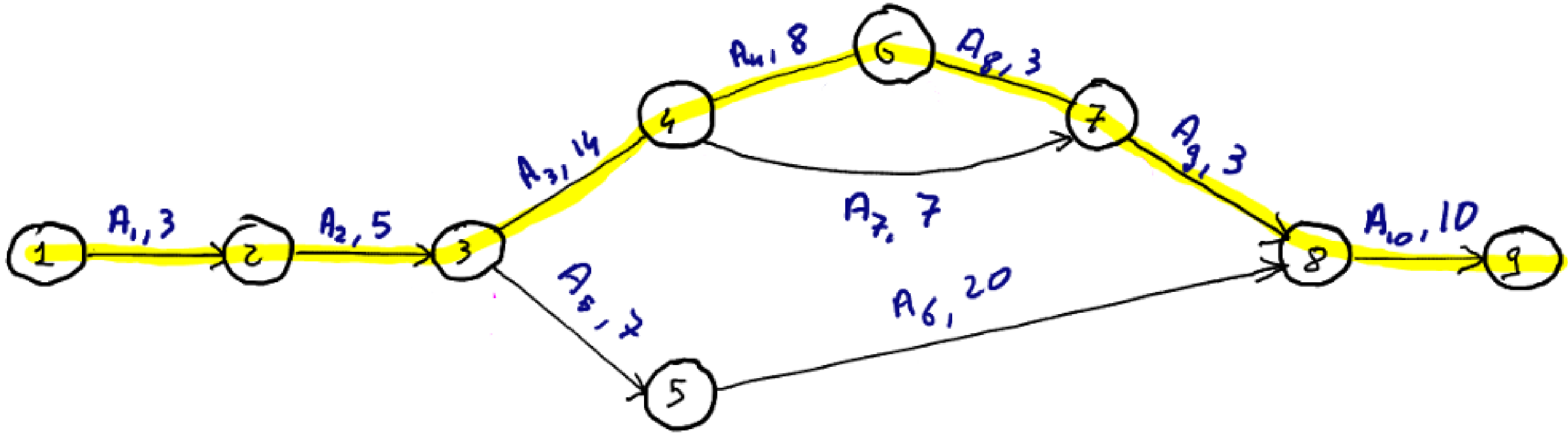
↳ To remember: $\alpha = 0.05 \rightarrow z_\alpha = 1.96$

↳ It is expected that D_c falls in I_α with probability 0.95

Sources of approximation in PERT

1) For different samples of the variables D_i, the critical path can change, while we assumed it does not

↳ there are heuristic methods for coping with this problem



2) Variables D_i are independent

Example

Activities A_i, $i = 1, \dots, 8$ verifying

$A_1 < A_3$ $\quad$ $A_2 < A_4$ $\quad$ $A_4 < A_7$

$A_5 < A_7$ $\quad$ $A_6 < A_8$ $\quad$ $A_7 < A_8$

$A_3 < A_6$ $\quad$ $A_3 < A_5$

Parameters a_i, m_i and b_i are given in the table

	a_i	m_i	b_i	μ_i	σ_i^2
A_1	1	2	3	2	$1/9$
A_2	2	3	4	3	$1/9$
A_3	1	2	3	2	$1/9$
A_4	2	4	6	4	$4/9$
A_5	1	4	7	4	1
A_6	1	2	9	3	$16/9$
A_7	3	4	11	5	$16/9$
A_8	1	2	3	2	$1/9$

$$\mu_i = \frac{a_i + 4m_i + b_i}{6}$$

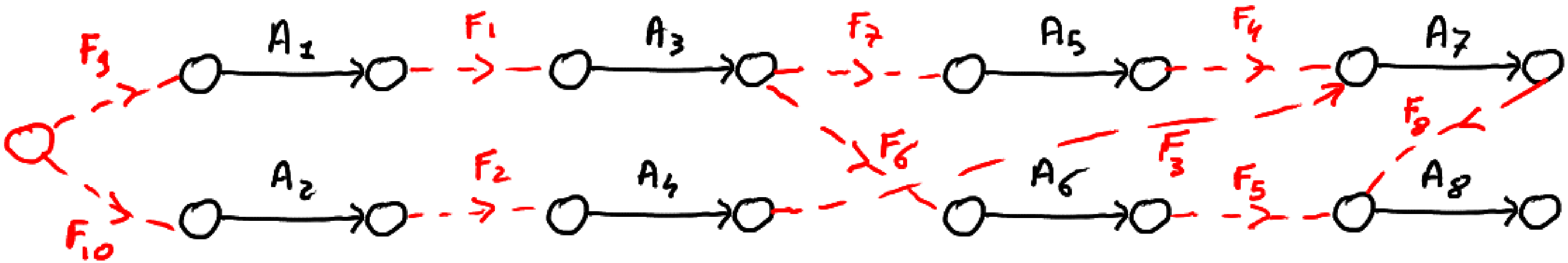
$$\sigma_i^2 = \left(\frac{b_i - a_i}{6}\right)^2$$

AOA network

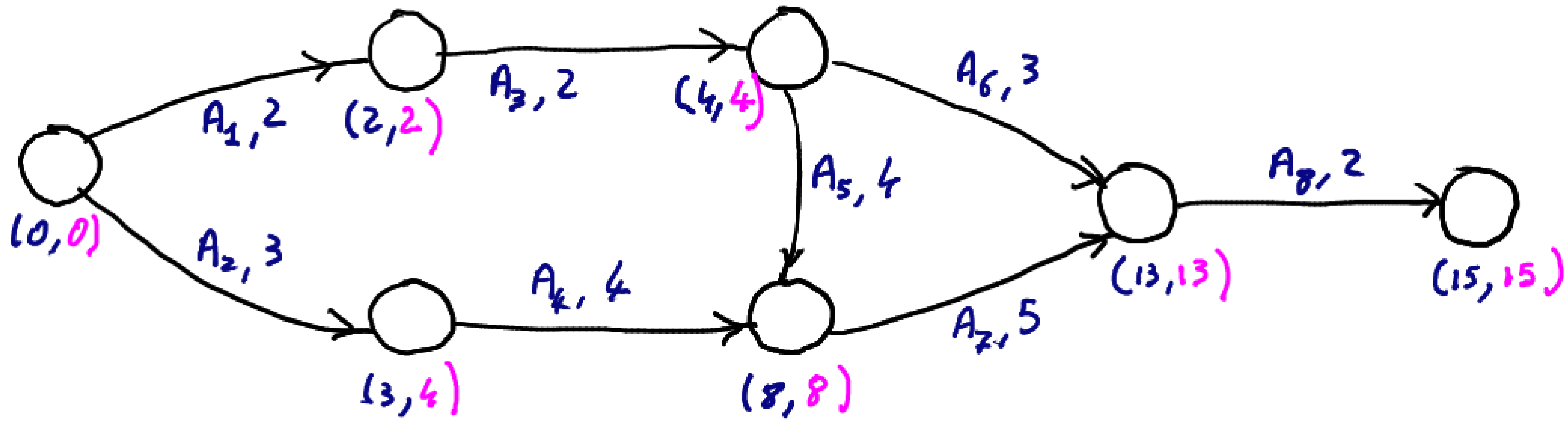
$A_1 < A_3$ $A_2 < A_4$ $A_4 < A_7$

$A_5 < A_7$ $A_6 < A_8$ $A_7 < A_8$

$A_3 < A_6$ $A_3 < A_5$



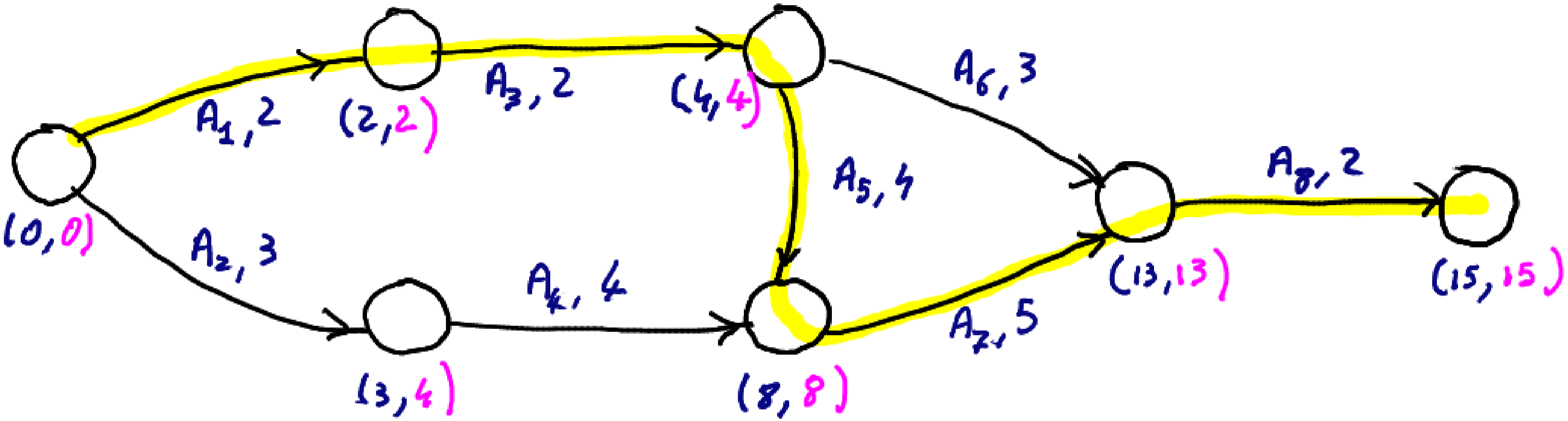
All dummy activities can be contracted (verify @ home)



Activity (i,j)	EST E_i	LST $L_j - d(i,j)$	Critical?
A_1	0	0	y
A_2	0	1	N
A_3	2	2	y
A_4	3	4	N
A_5	4	4	y

Activity (i,j)	EST E_i	LST $L_j - d(i,j)$	Critical?
A_6	4	10	N
A_7	8	8	y
A_8	13	13	y

PERT analysis



Critical path: A_1, A_3, A_5, A_7, A_8

Average project duration $\mu_c = 2+2+4+5+2 = 15$

Variance of project duration $\sigma_c^2 = \frac{1}{9} + \frac{1}{9} + 1 + \frac{16}{9} + \frac{1}{9} = \frac{28}{9} \rightarrow \sigma_c \simeq 1.76$

95% confidence interval for D_c

$$I_{0.05} = [15 - 1.96 \cdot 1.76\ ,\ 15 + 1.96 \cdot 1.76] = [11.55\ ,\ 18.45]$$